Size distribution and Sauter mean diameter of micro bubbles for a Venturi type bubble generator

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\textbf{A B S T R A C T}

An investigation is carried out to determine how air and water flow rates as well as the air inlet size (variable parameters) influence the size distribution of micro bubbles for a Venturi type bubble generator. Size distribution is extracted from the videos captured during the experiments using two different post processing algorithms which all report similar trends. The significance of the size distribution of micro bubbles is discussed and the results of the experiments are reported. It is shown that Sauter mean diameter and statistical parameters of log-normal distribution are influenced by the variable parameters and can be expressed using theoretical model based on dimensionless numbers containing these parameters. It is shown that such theoretical model reports influence of variable parameters on the investigated ones which is consistent with the population balance model as well as with the correlations previously reported in literature. Finally, the significance of correlation between variable parameters and statistical parameters of the distribution is discussed.

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1. Introduction

Micro bubbles are widely used in many industrial processes. Some examples include wine industry [1], waste water treatment [2–4] and growth enhancement of aerobic organisms [5]. Applicability of microbubbles in industrial processes results mainly from the following properties:

- large gas–liquid surface area;
- greater gas hold up; and
- slower rise velocity.

Size distribution of micro bubbles is the most important factor in determining the properties presented above. Smaller size of the bubbles increases gas–liquid interfacial area, which increases the rate of diffusion and hence the gas hold up. Smaller size also decreases bubble rise velocity. It is therefore necessary to understand how different parameters affect the size distribution of bubbles in order to be able to design an efficient bubble generator.

Various papers have been published describing the performance of specific bubble generators. Terasaka et al. [2] have described the performance of three different types of bubble generators for the purpose of waste water treatment: spiral liquid flow type generator designed by Ohnari et al. [6], Venturi type bubble generator and ejector type bubble generator. The comparison was made with reference to perforated plate and it was shown that the spiral liquid flow type generator achieved the best level of oxygen transfer coefficient. At the same time, since it required the greatest amount of power to run, the author suggested that the power consumption and performance should be considered together when evaluating the applicability of a given bubble generator for the desired industrial process.

Sadatomi et al. [7] invented a new bubble generator with a spherical body in a flowing liquid tube and tested its performance. The optimum ratio of the spherical body diameter to the tube diameter was found as well as the optimum axial position of the air suction holes. In addition, it was observed that the larger the bubble generator is, the better its performance.

Sadatomi and Kawahara [8] have then designed a new device with an orifice and a porous pipe instead of the spherical body and described its performance [9]. An optimum diameter ratio of orifice to pipe was found which maximises the ratio of bubble generation rate to power consumption rate. It was shown that the Sauter mean diameter of bubbles was about 12 \textmu m for a gas flow rate of 1 l/min and water flow velocity of 10 m/s at the nozzle. Furthermore, it was shown that under such conditions the size of the porous holes seemed to not affect the size of the bubbles.
In addition, both studies only considered the influence which variable parameters have on the average bubble size. Yet for many applications (for example waste water treatment), bubble size distribution is as important as the average size of the bubbles. However, none of the studies have investigated if a correlation can be found which describes the full shape of the distribution as a function of variable parameters.

In this paper an attempt is made to answer this question by investigating how water and air flow rates as well as air inlet size (variable parameters) affect the final size distribution of bubbles for a simple Venturi type bubble generator. About 50 different combinations of these parameters are investigated and two different post processing algorithms are used to reconstruct the size distribution for each case.

2. Experimental setup

2.1. Overview

In order to study the size distribution of bubbles, a Venturi type bubble generator was used. The device is illustrated in Fig. 1(a). It was placed in the circuit illustrated in Fig. 2(a). A Euromatic PVC 500 pump was used to pump the water from the tank and MotionBLITZ EoSenseCube 7 high speed camera was used to create videos of bubbly flow for further post processing.

Water flow rate through the visualisation cell was controlled using two valves as illustrated in Fig. 2(b). This allowed flow rate to vary from 0 l/min to 30 l/min. However, experiments were conducted for flow rate from 13 l/min to 30 l/min because at flow rate below 13 l/min large pocket of air was being formed at the top of the visualisation cell.

The bubble generator had replaceable air inlets of varying diameter in the range 0.2–1.5 mm. These are illustrated in Fig. 1(b). In the image the left air inlet has a diameter of 1.5 mm and the right air inlet has a diameter of 0.2 mm. Air flow rate was being controlled with a valve as illustrated in Fig. 2(a). Maximum achievable air flow rate was dependent upon inlet size. For the maximum air inlet of 1.5 mm in diameter it was equal to 1 l/min. However, for the purpose of visualisation a much lower air flow rate was used with the maximum value equal to 0.002 l/min. Such choice had to be made because visualisation cell had a width of 0.8 cm. For large air flow rate bubbles were being formed on all width levels and it was not possible to distinguish them in the video frames as, effectively, the whole picture was being painted black. Experimental conditions are summarised in Table 1.

Camera frame rate was set to 25 frames per second in order to ensure that each bubble could only appear on one frame. This elimi-
inated the need to track bubbles and hence simplified image post processing. Circular lamp with attached light diffuser was used as a light source. This resulted in the videos being brighter at the centre, but the quantity of light on the edges was still sufficient for post processing.

2.2. Videos post processing algorithm

2.2.1. Introduction

In order to obtain bubble size distribution from videos of bubbly flow it is necessary to utilise an algorithm capable of detecting air bubbles in water. Fig. 3 shows an example of captured frame with air bubbles. As can be seen, only a small fraction of bubbles of larger size are non-spherical. As the aim of current investigation is to examine the size distribution of microbubbles and because there are much more spherical small bubbles than large non-spherical, larger bubbles can be neglected. It is therefore possible to detect bubbles by finding circular objects in the movie frames.

Table 1

<table>
<thead>
<tr>
<th>Air flow rate, l/min</th>
<th>0.0003–0.002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water flow rate, l/min</td>
<td>13–30</td>
</tr>
<tr>
<td>Air inlet size, mm</td>
<td>0.2–1.5</td>
</tr>
</tbody>
</table>

![Circuit layout](image)

Fig. 2. Circuit layout.

![Example video frame](image)

Fig. 3. Example video frame.

2.2.2. Bubble detection using multi-layer background subtraction

This technique aims at a robust estimation of the radius even in the case of not perfectly circular bubbles, which is a very common situation during fast flows of liquids. The algorithm workflow accounts for two main stages: bubble detection and cluster splitting, respectively. Bubble detection is achieved by combining the Multi-Layer background subtraction algorithm (ML-BGS) [12] with morphological image processing operations. Once bubbles are detected, the second stage discriminates between single bubbles and bubble clusters. A bubble cluster is an agglomerate of bubbles located on different planes of the channel which appear as a unique overlapping group, due to similar bubble alignment with respect to the camera lens axis. Eventually bubble radii are indirectly estimated by computing bubble areas.

2.2.2.1. Bubble detection

The initial background subtraction employs the ML-BGS technique proposed in [12]. This algorithm was specifically designed to deal with cases of moving objects and fluctuating background colour, as shown in Fig. 4(a). The last aspect is important for our current investigation as the used light source produces a slightly varying background colour. In addition, due to the fact that light source emits radial light, a slight variation in the amount of light between the central part and the edges of the frame occurs. ML-BGS algorithm deals extremely well with both these complications, returning a gray-scale frame without the background, as shown in Fig. 4(b).

After having isolated the foreground image, an equalisation step allows areas of lower local contrast to gain a higher contrast, as shown in Fig. 4(c). Assuming that the image contains two classes of pixels following bi-modal histogram (foreground pixels and background pixels) the proposed method then calculates the optimum threshold separating the two classes so that their combined spread (intra-class variance) is minimal. The resulting thresholded image, as in Fig. 4(d), is further investigated in terms of its connected components using 8-connectivity Suzuki et al. [13], thus returning detected bubbles, as in Fig. 4(e). A step of reflex discovery and correction is performed before bubble cluster splitting. A reflex produces a sort of hole in the bubble which affects the estimation of its areas and, consequently, the quality of radius estimation. This problem is solved by checking the position of nested connected components, and filling the inner one with a unique white label. Finally, bubbles with radius less than three pixels are discarded from further processing, since they are considered as not relevant for further log-normal estimation and fitting.
2.2.2. Cluster splitting. Considering the bounding box of each detected connected component as a separated binary image, cluster splitting is performed by applying an empirically derived rule: if the area of the white connected component covers at least 70% of the rectangular bounding box, then the connected component is considered as a single bubble. Otherwise it is likely a bubble cluster which needs splitting.

This last situation occurs when multiple bubbles closely aligned with respect to the lens camera axis but on different planes partially occlude each other so that they are detected as a unique blob, as it happens for the top-right connected component in Fig. 4(e). In this case cluster splitting is achieved by adopting the steps described in Fig. 5.

The initial step recovers the foreground grayscale image obtained by ML-BGS for the candidate bubble cluster area, as shown in Fig. 5(a) to perform a local processing on cluster. After an equalisation step to locally enhance contrast (Fig. 5(b)), Circular Hough transform is performed to estimate the locations of single bubbles. Differently from previous approaches, the parameter choice has here been relaxed in order to obtain multiple bubble detections, as shown in Fig. 5(c). This guarantees to reduce the problem of parameter selection, a well-known defect of Circular Hough transform, especially in the case of non-perfectly circular bubbles, which is frequent in this application scenario.

After filling the estimated circles as in Fig. 5(d), the obtained image is again investigated in terms of its connected components using 8-connectivity, as in Fig. 5(e). Before estimating the final bubble radius, the same criterion as before is applied to check if found connected components are single bubbles or again bubble clusters. In this last case the procedure of cluster splitting is iteratively repeated until only single bubbles are obtained.

From all detected bubbles an estimation of the distribution of bubble radii is computed, as the one shown in Fig. 6. Notice that considered radii start from a minimum size around 90 µm, which corresponds to the minimum radius considered for the detected connected components, which has been set to three pixels.

2.2.3. Custom background subtraction followed by circular detection based on Circular Hough transform

In a second algorithm, a custom implementation of background subtraction was implemented.

As a first step, initial 50 frames were used to determine the background colour in each pixel. The median colour for these frames was stored in the matrix $C_{\text{background}}$. After this each frame of the movie was converted to black and white based on the difference between the colour of the given pixel and its median colour value as illustrated in Fig. 7(b). If the difference was found to be significant, the pixel was painted white indicating that it is part of the bubble. Otherwise, it was painted black. The difference was considered significant when the colour for the given pixel was less than 85% of the stored value. This threshold was determined manually and was tested on both lighter and darker regions in the frame. In both cases it performed well.

The need to account for the possibility of colour being less than median while still corresponding to background results from the fluctuations in the light source. Fig. 8 illustrates the typical colour
distribution in the given pixel for the first 50 frames. In most of those frames the colour of the pixel corresponds to the background colour and it can be seen that this value is around 150. However, it is clear that colour values close to 150 also correspond to background, unlike spikes around 110 and 90 which correspond to pixel being part of the bubble. In order to ensure that values close to the background colour were not reported as part of the bubble an 85% threshold had to be used.

Following background subtraction, image was analysed and connected white components were filled with white colour as illustrated in Fig. 7(c). This needed to be done to ensure that inner part of the bubble was not left black which could happen for larger bubbles having a bleak inside of them. Bubbles were then identified using a function based on Circular Hough transform. During post processing, gradient threshold was set to 0.001; radius of the filter used in the search of local maxima in the accumulation array was set to 10; and the detection of concentric circles was disabled.

While the detection was good for isolated bubbles, in the more complex scenario of the overlapping bubbles or bubbles of irregular shape the detection could fail either by detecting several small bubbles as a single large bubble or by detecting part of the irregularly shaped bubble as a small bubble. It was observed that such cases did not correspond to any specific bubble size – the overlapping occurred because bubbles located on different planes of the channel could become aligned with the camera lens axis. Hence, in this case, differently from the first algorithm, it was decided that by discarding these cases the overall distribution would not be influenced provided that enough frames were used to construct it.

In order to filter out the overlapping bubbles two tests were performed on the square regions surrounding bubble (with sides equal to bubble diameter):

1. Average colour within the region corresponding to $r_1$ from Fig. 9 was computed. If it was more that 40% black then it meant that the detected bubble is actually a group of bubbles and such result was discarded.

2. Average colour within the region corresponding to $r_2$ from Fig. 9 was computed. If the colour within the region was more than 72% white such bubble was discarded as it meant that the algorithm has detected only a portion of the bubble (or groups of bubbles).

The threshold value for each of these tests was determined manually by looking at the cases of wrong detection and inspecting the colour value within both regions. After the threshold values were chosen they were tested on many frames and were found to perform well.

All detected bubbles (illustrated in Fig. 7(d)) passing these tests were stored and later used to construct the PDF distribution of bubble sizes. Typical PDF for the custom algorithm is represented in Fig. 10. As can be seen, current implementation of Circular Hough transform was not able to detect bubbles with diameter less than 6 pixels (which corresponds to 90 μm). However, the detected PDF was good enough to be able to obtain log normal fit.

3. Results and discussion

Several movies (50) were analysed with different air inlet sizes, water and air flow rates. In order to study the size distribution of bubbles it was decided to investigate the influence each parameter has on the Sauter mean diameter and on parameters $M$ and $S$ of the

**Fig. 6.** Estimated distribution of bubble radii (blue bars) with over imposed fitted log-normal distribution (red line) of one the analysed video. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
log normal distribution. Sauter mean diameter can be defined using Eq. (1) for a discrete distribution.

$$d_{32} = \frac{\sum_{i=1}^{n} d_i^3}{\sum_{i=1}^{n} d_i}$$  \hspace{1cm} (1)

In the case of current experiments detected radius range is limited by a cut off value. It is therefore better to avoid using Eq. (1) and, instead, calculate Sauter mean diameter from the estimated parameters of Log Normal distribution. Log Normal distribution (\(P(x)\)) can be defined using Eq. (2) and Sauter mean diameter can be defined using Eq. (3).

$$P(x) = \frac{1}{S\sqrt{2\pi}} e^{-\left(\ln x - M\right)^2/2\sigma^2}$$  \hspace{1cm} (2)

$$d_{32} = e^{M - 2.5\sigma^2}$$  \hspace{1cm} (3)

While Sauter mean diameter gives a good representation of a “typical bubble” size, in the present study the influence each parameter has on \(M\) and \(S\) from Eq. (2) is investigated as well.

### 3.1. Sauter mean diameter

#### 3.1.1. Results

Fig. 11 shows the reported size of the bubbles for each algorithm. As can be seen from Fig. 11, all algorithms reported very similar Sauter mean diameters and the change in the Sauter mean diameter with the change in water flow rate. At the same time it is possible to see that, on average, ML-BGS algorithm reported higher values of Sauter mean diameter compared with the Custom algorithm. This difference can be attributed to the use of different thresholding techniques. While thresholding increases the uncertainty in the measurements, its use is, firstly, unavoidable, and, secondly, should not change the reported trend. Instead, it is expected that thresholding will affect the constant of proportionality for fitted models. However, as the main purpose of this study is to answer the question whether size distribution of bubbles can be described with the correlation that depends on variable parameters, it is possible to conclude that different constants of proportionality do not present a problem.

Fig. 11 contains values from all tested cases and therefore reports values for cases with all three investigated parameters changing. Therefore, even though Fig. 11 indicates that there is a strong correlation between water flow rate and Sauter mean diameter it cannot be used alone to draw such conclusion. In order to investigate correctly how different parameters affect the size distribution it is therefore necessary to obtain model fit for each algorithm.

During the experiments the only variable parameters were air inlet size, water flow rate, air flow rate and static pressure at air inlet. Static pressure is a dependent parameter and is determined by water flow rate and outlet pressure at the pump. It was also found that during the experiments suction pressure was always in the range 6–7 kPa. It was therefore decided not to treat it as an independent variable.

Remaining parameters can be combined into different dimensionless numbers (Reynolds number, Weber number, air–water fraction, etc.). Kolmogorov [14] and Hinze [15] have shown that a maximum stable bubble size in gas–water mixtures is inversely proportional to turbulent energy dissipation rate which, in turn, is proportional to Reynolds number of the flow. In addition, Gabbard [11] has shown that volume averaged bubble diameter depends on the Reynolds number of the flow. Therefore, it was decided that water Reynolds number must be used in the model.

Gabbard [11] has also shown that size of the bubbles depends on the surface tension. However, in the present study surface tension was constant (ignoring minor changes due to minimal temperature differences between measurements). It was therefore decided that including dimensionless numbers which contain surface tension terms would imply that a correlation was found which does take this relationship into account without it actually being a case. Therefore, it was decided to not include Weber, Bond and Capillary numbers into the model. In addition, using similar logic,
it was decided to ignore any other dimensionless numbers which were constant during the present experiments (for example viscosity ratio).

Kawashima et al. [10] have indicated that there is a strong possibility that air–water ratio influences the size distribution of bubbles. It was decided to include this dimensionless number into the model. After this the only variable parameter which wasn’t accounted for in the model was the air inlet size. In order to account for it was decided to include air Reynolds number based on the air inlet into the model. This also allowed to separate the influence which increased air concentration and initial air velocity have on the size of the bubbles.

The final theoretical model under investigation is described by Eq. (4):

$$
\frac{d_{12}}{d_0} = aRe_w Re_a \alpha
$$

where $d_{12}$ is the Sauter mean diameter, $d_0$ is the diameter of air inlet, $Re_w$ is water Reynolds number, $Re_a$ is air Reynolds number (based on air inlet) and $\alpha$ is air/water ratio.

Table 2 shows the obtained model fits for different algorithms together with the obtained $R^2$ values. These results were obtained using Matlab’s function which estimates coefficients for nonlinear regression. As can be seen, all algorithms reported very decent $R^2$ values and fairly similar coefficients. Moreover, it can be seen that both algorithms reported similar power coefficients. At the same time the multiplication constant is different, but this can be explained by the use of slightly different thresholding which influences the reported diameter of each bubble.

In order to comment on the influence of independent parameters on the size distribution of bubbles, data in Table 2 needs to be transformed to show how each of these parameters is incorporated into dimensionless numbers.

To achieve this, Eq. (4) can be stripped of all constants and converted to Eq. (5).

$$
d_{12} \sim \frac{aRe_w d_0^{0.8}}{d_{12}^{1.2}}
$$

It may also be useful to introduce another representation in which air flow rate is removed as independent variable and is substituted instead (with the account for correct power) by air/water fraction. Such conversion is illustrated in Eq. (6).

$$
d_{12} \sim \frac{Q_b}{Q_w} \frac{d_0^{1-C}}{d_{12}^{2-C}}
$$

Results of these conversions are represented in Table 3.

### 3.1.2. Discussion

As can be seen from Table 3, all algorithms show that there is a strong dependence of the bubble size on water flow rate. Even considering non ideal fits of each model it is possible to conclude that increase in the water flow rate leads to a decrease in bubble size and this influence is much stronger than the influence of any other parameter. All theoretical models also show that an increase in air/water ratio leads to an increase in the bubble size. Finally, all algorithms show that air inlet size is either proportional to the bubble size or has negligible influence.

Gabbard [11] has proposed a correlation according to which volume averaged bubble size would be proportional to liquid velocity to the power of −1.2. He then conducted tests and found that the power constant for his Venturi type bubble generator was −0.8 instead of −1.2. In the present study, Sauter mean diameter was analysed instead of the volume averaged diameter. However, in light of the existent correlation it was decided to compare the influence which presented algorithms would have on the volume averaged diameter of the bubbles. This influence is summarised in Table 4. Presented results were obtained using the model described in Eq. (4), $R^2$ for both algorithms was greater than 0.95. The values obtained in the current investigation are clearly different to those reported by Gabbard [11]. This difference is likely to be a result of several factors.

Firstly, Gabbard [11] has used conical bubble generator which will have a different turbulent energy dissipation rate to the one which was used in the current study. Secondly, current study did not ignore the influence which flowing gas has on the size of the bubbles. This enabled authors to obtain higher $R^2$ values and to eliminate the need to ignore some measured points (as was done by Gabbard [11]). In fact, when a model was analysed which only takes into account the influence which Reynolds number of water has on the volume averaged size of the bubbles, the reported power constant was −1.3 with an $R^2$ value of 0.3. This shows that the reported influence can be very different depending on how many parameters are taken into account. Unfortunately, Gabbard [11] did not present any goodness of fit data for his correlation so it is impossible to make a direct comparison. Nevertheless, we believe that the correlation obtained by Gabbard [11] is significant and does not contradict the findings of the current study.

In order to further comment on the obtained results it is important to recall that movies were taken downstream of the air inlet. This implies that the size distribution of bubbles was evolving from the inlet point to the point where movies were recorded. This evo-

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### Table 3

<table>
<thead>
<tr>
<th>Independent parameter</th>
<th>ML-BGS</th>
<th>Custom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{12}$ ~ $Q_w^{0.9517}Q_b^{0.2496}d_0^{0.0747}$</td>
<td>$d_{12}$ ~ $Q_w^{0.3739}Q_b^{0.0561}d_0^{0.0747}$</td>
<td></td>
</tr>
<tr>
<td>$d_{12}$ ~ $Q_w^{0.7202}Q_b^{0.2454}d_0^{0.047}$</td>
<td>$d_{12}$ ~ $Q_w^{0.6977}Q_b^{0.047}d_0^{0.0561}$</td>
<td></td>
</tr>
</tbody>
</table>

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### Table 2

<table>
<thead>
<tr>
<th>Fitted model parameters</th>
<th>Sauter mean diameter</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML-BGS</td>
<td>$\frac{d_{12}}{d_0} = 1215.9Re_w^{1.796}Re_a^{0.5110}$</td>
<td>0.941</td>
</tr>
<tr>
<td>Custom</td>
<td>$\frac{d_{12}}{d_0} = 1294.2Re_w^{1.772}Re_a^{0.047}d_0^{1.0156}$</td>
<td>0.932</td>
</tr>
</tbody>
</table>
The relationship between $d_{\text{max}}$ and $d_{32}$

It is well known from literature that the maximum bubble diameter is related to the Sauter diameter with coefficient of proportionality between 1.5 and 3 [18]. Due to this it was decided to investigate such relationship for the two image processing algorithms used in this study.

However, as was noted before, in the present study maximum detectable bubble diameter was deliberately cut in order to improve algorithm performances. So, instead of using the obtained value directly, it was decided to consider maximum bubble diameter to be equal to the diameter at 99% percentile of the corresponding log-normal distribution. Fig. 12 illustrates the obtained relationship between $d_{\text{max}}$ and $d_{32}$.

Because data from both algorithms are very close to each other, it was decided to find a single relationship between $d_{\text{max}}$ and $d_{32}$. Such relationship is described by Eq. (10).

$$d_{\text{max}} = 1.6432 d_{32}$$

$R^2$ value for such fitting was found to be 0.975, which shows that the obtained model fits data very well and is in a good agreement with values previously reported in literature (for example Azzopardi and Hewitt [18]). Furthermore, it shows that the obtained coefficient of proportionality is suitable for both algorithms. Overall, this further implies that both algorithms reported meaningful results.

3.3. Statistical parameters

3.3.1. Results

An investigation was carried out to determine how variable parameters affect lognormal parameters $M$ and $S$ from Eq. (2). To do so a model similar to that described in the previous section was proposed for each statistical parameter. The investigated model for $M$ and $S$ from Eq. (2) thus have a form described by Eqs. (11) and (12).

$$M = aRe^b Re^c x^d$$

$$S = fRe^e Re^g x^k$$

Results of the fitted model are summarised in Table 5.

![Fig. 12. Relationship between $d_{\text{max}}$ and $d_{32}$.](image-url)
3.3.2. Discussion

Firstly, as can be seen from Table 5, each algorithm produced a very good $R^2$ estimate for the fitted model which suggests that $M$ and $S$ can indeed be represented using the proposed model. This is an important result as it means that the results for the given algorithm can be predicted from the theoretical model. It also means that statistical parameters strongly depend on the experimental conditions, which implies that the difference in the results is not due to some random errors. More importantly, it indicates that correlation exists not only for the average bubble size but also for the log-normal distribution of bubbles. As was discussed earlier, this result is extremely important for many industrial applications.

Secondly, it is clear that each algorithm demonstrated the same principal influence of each variable parameter on $S$ (i.e. Reynolds numbers, air–water ratio, etc.). From Table 5 it follows that an increase in $R_{av}$ and $Re_a$ reduces the parameter $S$ while an increase in air–water ratio increases $S$. However, algorithms do not report such a uniform influence of parameters on $M$.

In order to simplify the discussion of the results it is helpful to refer to Eqs. (13) and (14) to be able to comment on the influence of parameters on the mean and variance of the distribution.

$$
\mu = e^{M - S^2/2} \tag{13}
$$

$$
\sigma^2 = e^{2M - 3S^2} (e^{S^2} - 1) = \mu^2 (e^{S^2} - 1) \tag{14}
$$

Both mean and variance are functions of $M$ and $S$ and it is therefore non-trivial to comment on the influence that variable parameters have on these. However, from Eq. (14) it follows that coefficient of variation (defined by Eq. (15)) depends only on $S$. And since all algorithms reported identical influence of variable parameters on $S$ it is possible to discuss how they influence the coefficient of variation.

$$
c_v = \frac{\sigma}{\mu} = \sqrt{e^{S^2} - 1} \tag{15}
$$

Coefficient of variation decreases when water Reynolds number increases. This can happen with the increase in water flow rate, which will also reduce the air–water ratio. As the former is proportional to coefficient of variation, the net result will also be a decrease in the coefficient of variation. This is consistent with the population balance model as for higher turbulence levels the maximum stable bubble size will be reduced and hence more bubbles will have size closer to the mean value.

Similarly, with the increase in the air Reynolds number coefficient of variation gets reduced with other parameters staying constant. This can happen if air–water ratio and Reynolds number of water stay constant. This is possible if, for example, air inlet size gets reduced while all the flow rates are kept the same. It is therefore possible to conclude that lower air inlet size forces the distribution to have a more profound peak.

Finally, increase in the air–water ratio increases the coefficient of variation for equal air and water Reynolds numbers. The isolated influence of increased air–water ratio can be achieved by increasing air flow rate while also increasing inlet size to make the inlet air velocity (and hence air Reynolds number) stay constant. In this case coefficient of variation increases meaning that diameters are more spread out. This is also consistent with population balance model as increased air flow rate leads to an increased concentration of bubbles which, in turn, allows for higher rate of bubble coalescence. This increase in the rate of coalescence holds also true for bubbles of mean size as well, which obviously leads to greater variability in the size of the bubbles.

4. Conclusions

An experimental investigation was conducted in order to study the influence of different parameters on the size distribution of bubbles in a Venturi type bubble generator. Influence of air inlet size as well as air and water flow rates were investigated.

Two different algorithms were used to extract the size distribution of bubbles from video recordings. Theoretical model was fitted for Sauter mean diameter for each algorithm with a good $R^2$ value. This correlation was compared with those previously reported in the literature and it was found that it agrees with them.

Furthermore, it was found that a correlation exists which describes how variable parameters affect the statistical parameters of log-normal distribution with a good $R^2$ value for all algorithms. This is a very significant result as for many practical applications knowing the average bubble size is not enough – it is equally important to know what proportion of bubbles will have a certain size.

In addition to the practical benefit, the existence of such correlation makes it possible to conduct tests on the post processing algorithms. With further development of computers and video recording devices the attractiveness of this size measurement technique becomes greater while the cost of it becomes lower. However, it is important to be able to test the algorithm against known reference points to be able to judge its applicability. By knowing that a correlation exists that can describe which size distribution the algorithm will report for a given condition it is possible to conduct analysis only for portion of reference points and then make conclusions about the remaining size distributions based on correlation. This can vastly improve the speed of development of new algorithms (especially those which require a lot of computational power) which, ultimately, can make the described measurement technique even more attractive.

Finally, each theoretical model has shown that the size of the bubbles is inversely proportional to the water flow rate which is consistent with the population balance model. It was also shown that the increase in the air-water ratio results in the increase of the bubble size. The influence of air inlet size could not be determined certainly due to the difference in the reported influence
from different algorithms, although a strong possibility of this conclusion being valid was discussed. At the same time it was shown that the coefficient of variation gets reduced with the increase in the water Reynolds number, gets reduced with the increase in air Reynolds number and gets increased with the increase in air–water ratio.

So, in addition to results reporting dependence of size distribution on air and water flow rates, they also clearly indicate that these parameters can significantly alter the size of the bubbles. This leads to two significant conclusions:

1. The size of the bubbles can be altered/controlled during the operation of the device.
2. In order to describe the performance of the bubble generator one needs to consider different conditions.

First conclusion implies that it might be possible to optimise the performance during the operation of the device. As a simple example one can imagine a bubble generator capable of producing very fine micro bubbles being used for the application where the size of the bubbles can be greater without compromising the desired objectives. In such a case water flow rate could be lowered in order to still produce the desired level of performance while using a lower amount of energy.

Second conclusion implies that when designing a new bubble generator it is important to analyse its performance under various suitable conditions. For example, a device designed to clean waste water may perform differently depending on how deep it is submerged as the increase in outlet pressure may result in the significant decrease of the water flow rate and hence influence the size distribution of bubbles.

References


